

Plasma shaping effects on NTM threshold physics

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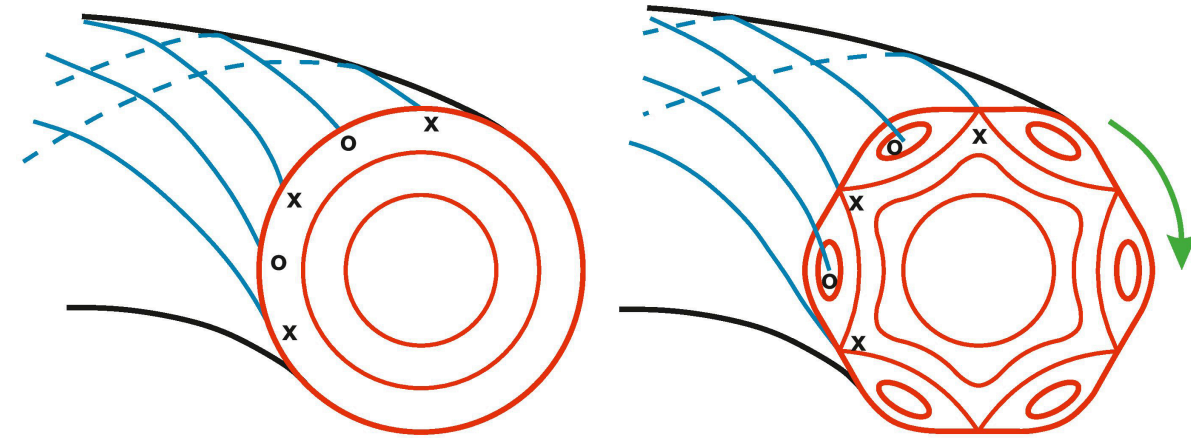
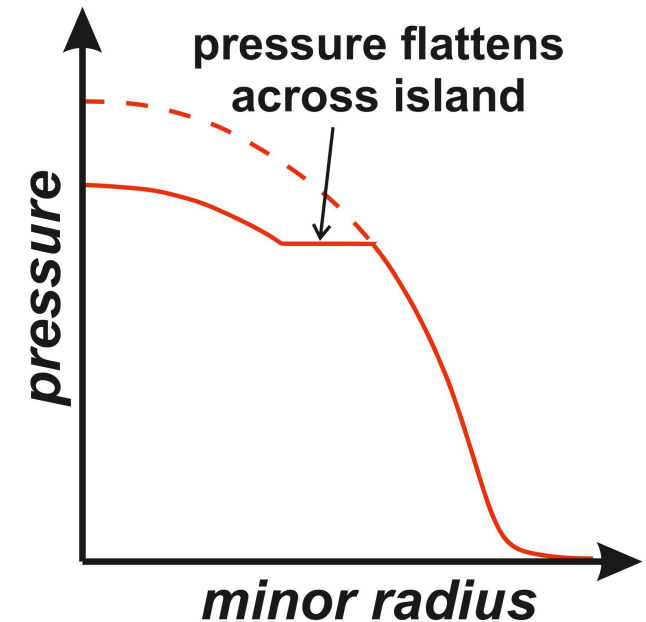
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Neoclassical tearing modes: why is it important?

- large scale MHD instabilities that arise due to filamentation of current density in plasma
- magnetic island is amplified by bootstrap current filamentation
- pressure gradient inside the island is reduced due to enhanced particle and heat transport in radial direction
- fusion power output is reduced
- NTMs can lead to plasma disruptions



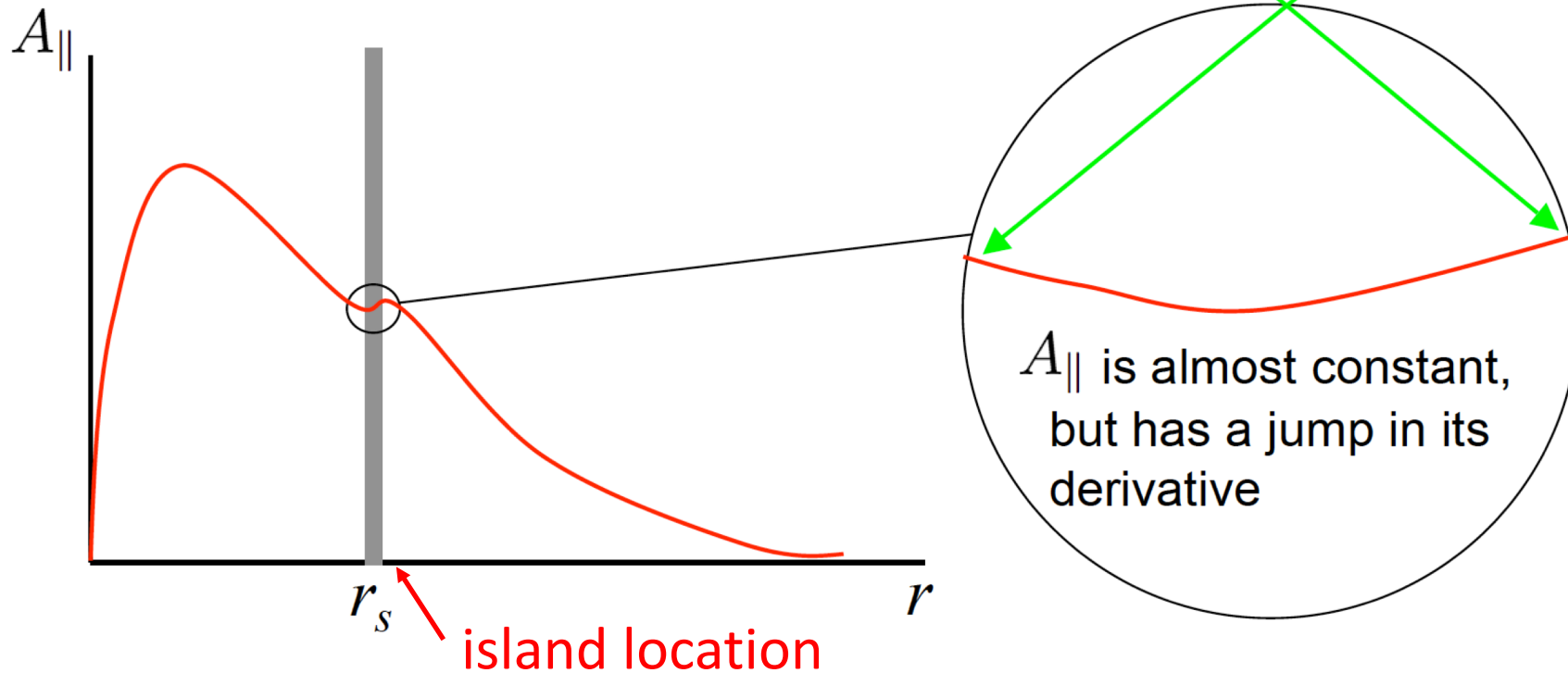
Contributions to NTM growth

- Ampere's law (schematically)

$$[\nabla \times \delta \mathbf{B}] \cdot \mathbf{b} = -\frac{\partial^2 A_{\parallel}}{\partial r^2} = \delta J_{\parallel} = \sigma \frac{\partial A_{\parallel}}{\partial t} + J_{loc}$$

Induction

Neoclassical contributions, including bootstrap current hole



$$\Delta' = \frac{1}{A_{\parallel}} \left. \frac{\partial A_{\parallel}}{\partial r} \right|_{r_s-w}^{r_s+w}$$

(Determined by global physics, comes from ideal MHD)

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Induction ↑

Neoclassical contributions, including bootstrap current hole

- We integrate across the island to obtain

$$\Delta' A_{\parallel} = \int \left(\sigma \frac{\partial A_{\parallel}}{\partial t} + J_{loc} \right) dr$$

$$\Delta' = \frac{1}{A_{\parallel}} \frac{\partial A_{\parallel}}{\partial r} \Big|_{r_s-w}^{r_s+w}$$

(Determined by global physics, comes from ideal MHD)

- Island width w is proportional to $A_{\parallel}$ and thus

$$\frac{\tau_r}{r_s^2} \frac{dw}{dt} = \Delta' + \Delta_{loc}$$

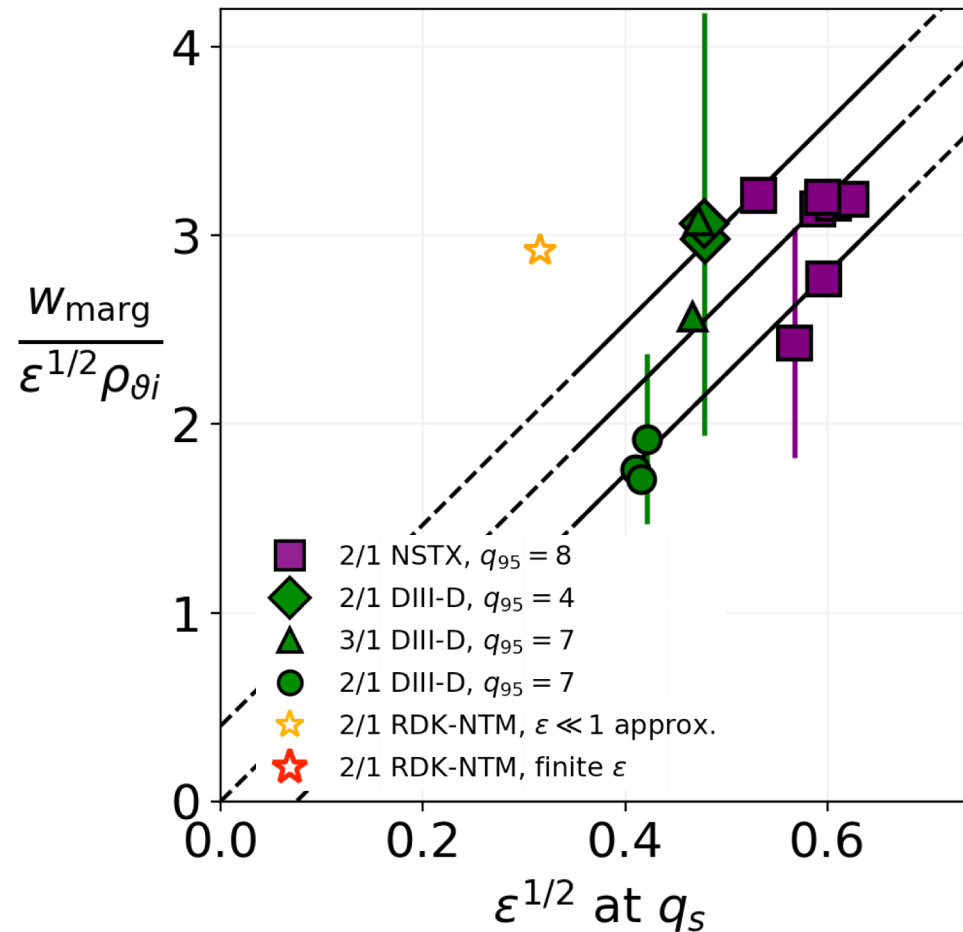
Layer theory

$$\Delta_{loc} = -\frac{1}{2A_{\parallel}} \int d\psi \int d\xi \delta J_{\parallel} \cos \xi$$

NTM threshold and stabilisation by ECCD

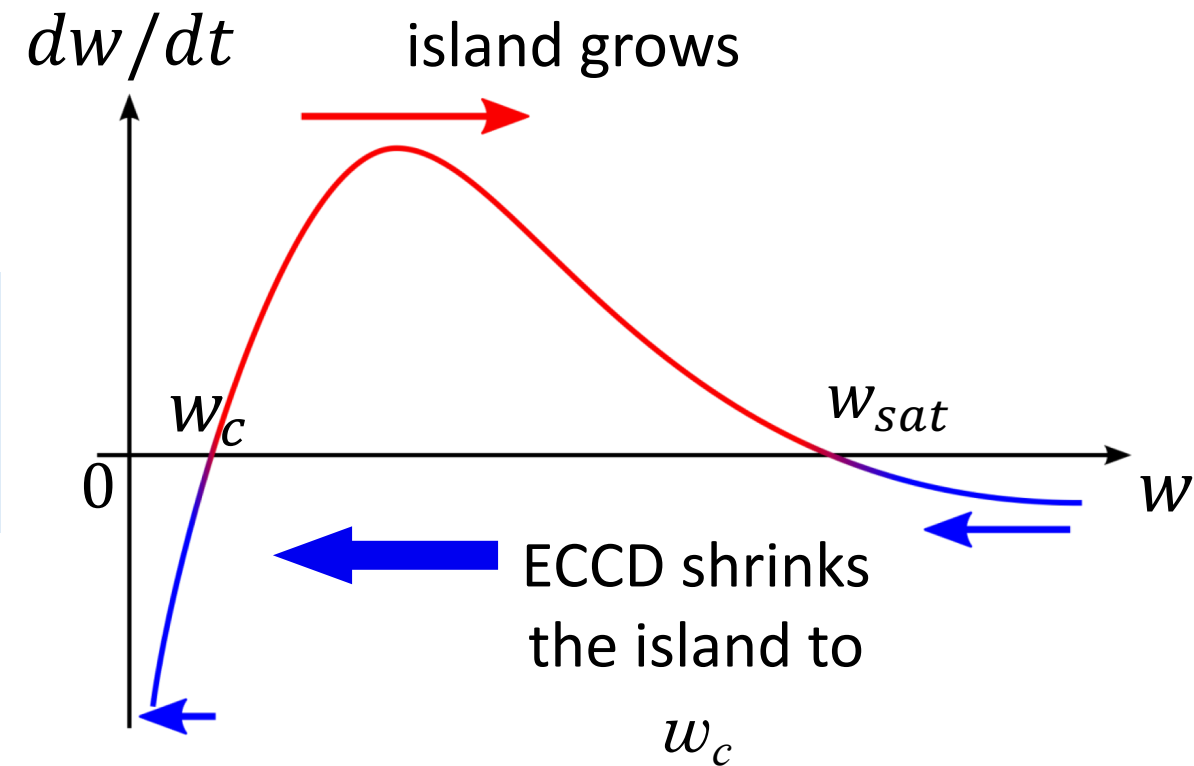
- Replace lost bootstrap current inside island with ECCD
- Below threshold width, island stabilises

Experimental prediction:



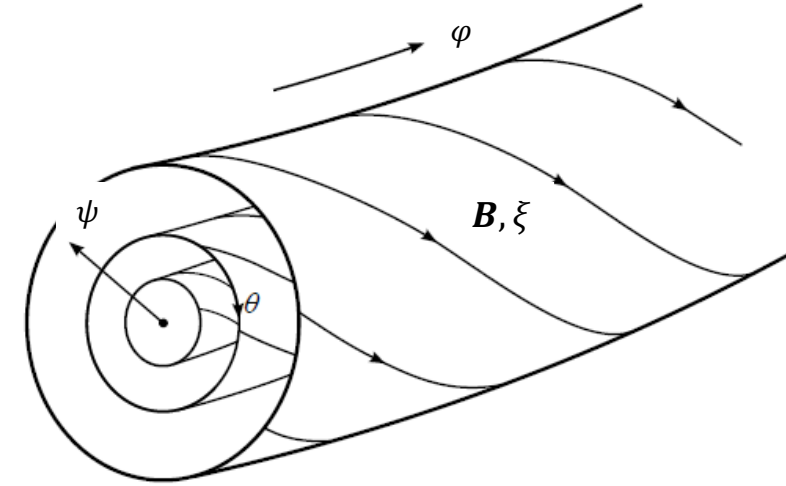
To develop a predictive model of the NTM threshold width w_c

- requires $\Delta_{loc} = -\frac{1}{2A_{||}} \int d\psi \int d\xi \delta J_{||} \cos \xi$
- need to consider small islands, comparable to ion banana orbit width
- hence, need to use kinetic theory at this length scale



Theoretical background

- finite beta finite aspect ratio tokamak
- equilibrium quantities localised to magnetic island resonant surface
- island width, $w \sim$ banana orbit width, $\rho_b \ll$ minor radius, a
expand in $\Delta = w/a \ll 1$
- solve **drift kinetic equation** for ions with momentum conserving collision operator up to $O(\Delta g)$,
 g is perturbed distribution function
- solve **drift kinetic equation** for electrons, coupled to ions
extends E. Poli *Phys. Rev. Lett.* 88 7 (2002) 075001
- solution consistent with field equations; electrostatic potential determined from quasi-neutrality



dimensionality reduction:

$$\{\psi, \xi, \vartheta, \lambda, V; \sigma\}$$



- if in terms of toroidal angular momentum

$$\{p_\varphi, \xi, \lambda, V; \sigma\}$$

$$p_\varphi = p_\varphi(\psi, \vartheta, \lambda, V)$$



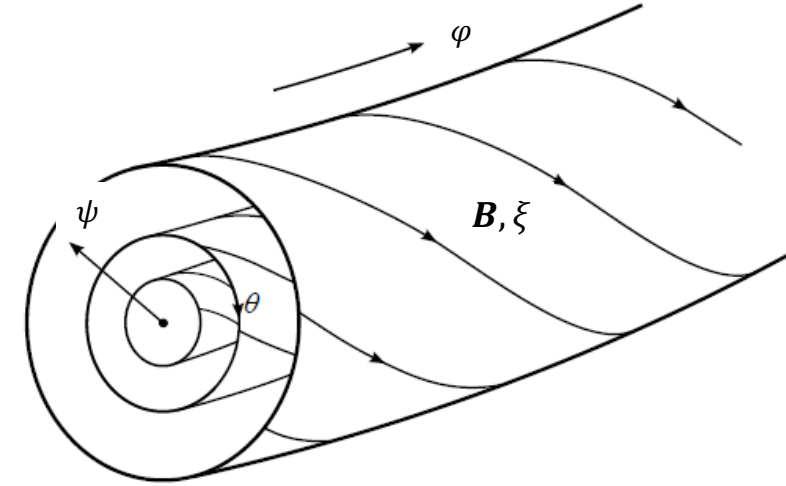
- small collisions $<$ free streaming

$$\{S, \lambda, V; \sigma\}$$

$$S = S(p_\varphi, \xi, \lambda, V)$$

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Dimensionality reduction

- Split distribution function into equilibrium piece and perturbation related to the NTM island

$$f_i = \left(1 - \frac{e\Phi}{T}\right) F_M(\psi = 0) + G(\psi, \xi, \theta, \lambda, v, \sigma)$$

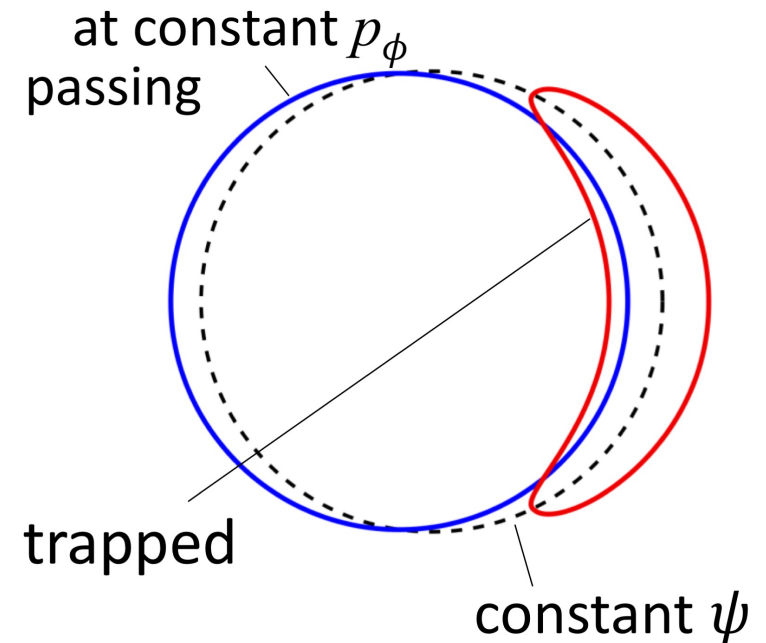
$$G = \sum_k G_k \left(\frac{w}{r}\right)^k$$

- To leading order

$$\left. \frac{v_{||}}{Rq} \frac{\partial G_0}{\partial \theta} \right|_{p_\phi} = 0 \quad \Rightarrow \quad G_0(\psi, \xi, \theta, \lambda, v) = G_0(p_\phi, \xi, \lambda, v)$$

$$p_\phi = \psi - \rho_\theta v_{||}$$

- Reduces the problem to 4D



Orbit averaged equation: 4D model

- Proceed to next order in Δ to eliminate G_1
- Provides a 4D orbit-averaged drift kinetic equation for G_0

Free streaming
along unperturbed field

Magnetic drift

ExB drift
toward X-point

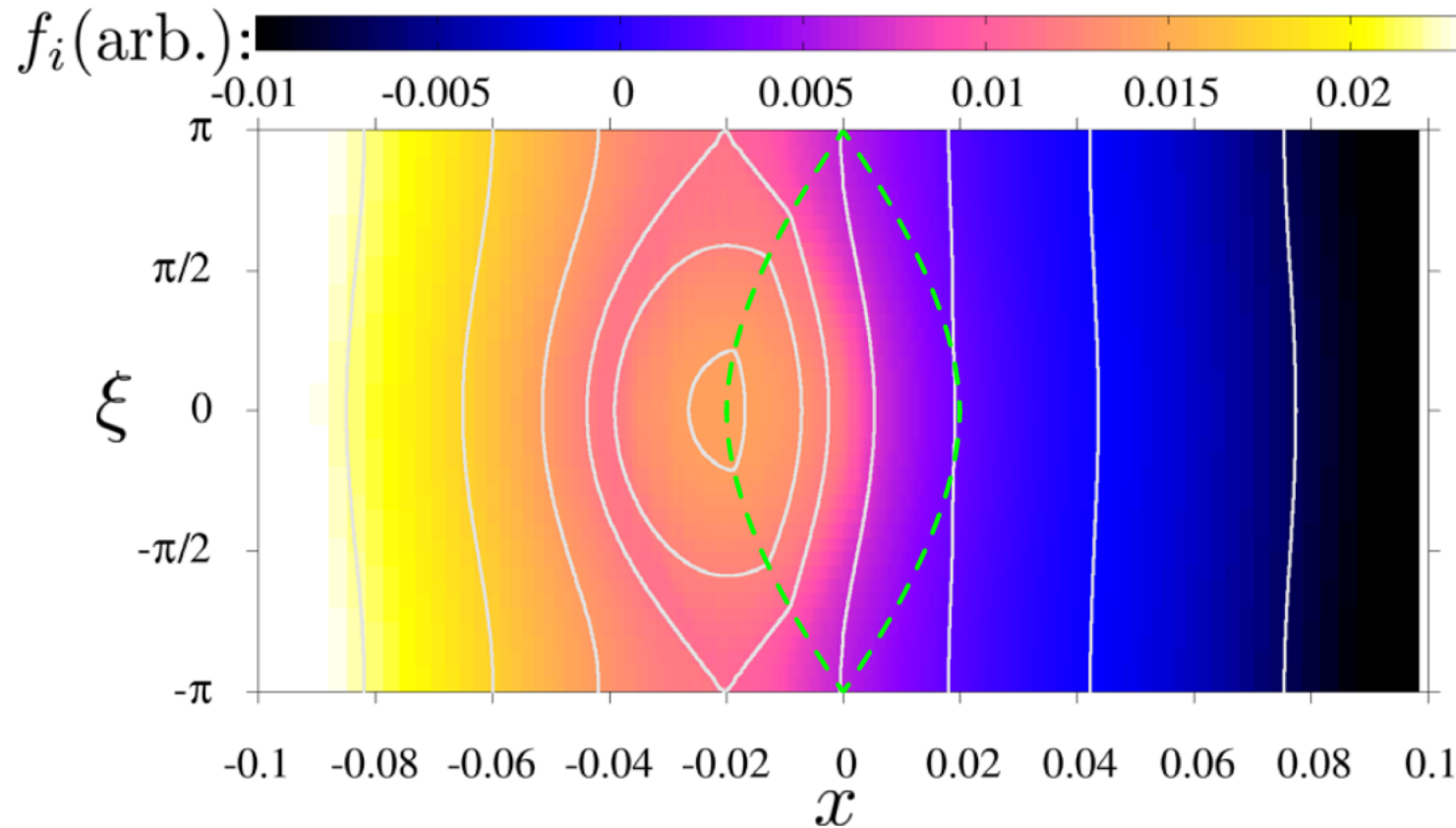
$$\left[\frac{\hat{w}}{\hat{L}_q} \hat{p}_\varphi \cdot \Theta(\lambda_c - \lambda) - \hat{\rho}_{\vartheta j} \hat{\omega}_D - \frac{\partial}{\partial \hat{p}_\varphi} \bigg|_{\xi, \vartheta} \frac{1}{2} \left\langle \frac{\hat{\rho}_{\vartheta j}}{\hat{V}_\parallel} \hat{\Phi} \right\rangle_{\vartheta}^{p_\varphi} \right] \frac{\partial g_j^{(0)}}{\partial \xi} \bigg|_{p_\varphi, \vartheta, \lambda, V; \sigma} +$$

$$+ \left[\frac{\partial}{\partial \xi} \bigg|_{p_\varphi, \vartheta} \frac{1}{2} \left\langle \frac{\hat{\rho}_{\vartheta j}}{\hat{V}_\parallel} \hat{\Phi} \right\rangle_{\vartheta}^{p_\varphi} - \frac{1}{4} \frac{\hat{w}}{\hat{L}_q} \sin \xi \cdot \Theta(\lambda_c - \lambda) \right] \frac{\partial g_j^{(0)}}{\partial \hat{p}_\varphi} \bigg|_{\vartheta, \xi, \lambda, V; \sigma} = \tilde{C}_j,$$

Radial ExB drift

radial
streaming
along
perturbed field
of island

Pitch angle
scattering and
momentum
conservation



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Colour contour plots of ion distribution function in (ψ, ξ) plane.

Dashed line: magnetic island separatrix

Solid lines: contours of constant S

Low collisions: ion distribution reduces to a function of S

3D solution: Drift islands

- 4D problem in terms of $\{p_\varphi, \xi, \lambda, V; \sigma\}$
- is equivalent to 3D when written in terms of $\{S, \lambda, V; \sigma\}$

- S introduces drift islands explicitly:

$$S = \frac{\hat{w}}{4\hat{L}_q} \left[2 \left(\hat{p}_\varphi - \sigma \frac{\hat{\omega}_D \hat{\rho}_{\vartheta j} \hat{L}_q}{\hat{w}} \right)^2 - \cos \xi \right] \Theta(\lambda_c - \lambda) - \sigma \hat{\omega}_D \hat{\rho}_{\vartheta j} \hat{p}_\varphi \Theta(\lambda - \lambda_c) - \frac{1}{2} \left\langle \frac{\rho_{\vartheta j}}{\hat{V}_\parallel} \hat{\Phi} \right\rangle_{\vartheta}^{p_\varphi}$$

passing trapped

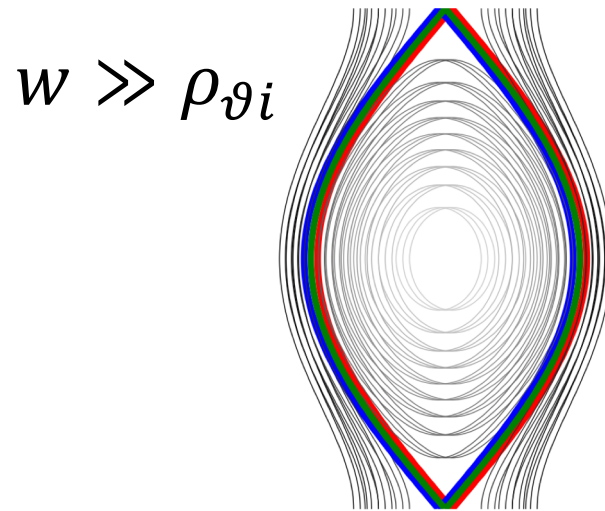
- radial shift is proportional to ρ_ϑ
- radial shift depends on $\sigma = V_\parallel / |V_\parallel|$, i.e. magnetic island is localised between $\sigma = +/-$ drift islands
- distribution function is flattened across drift islands

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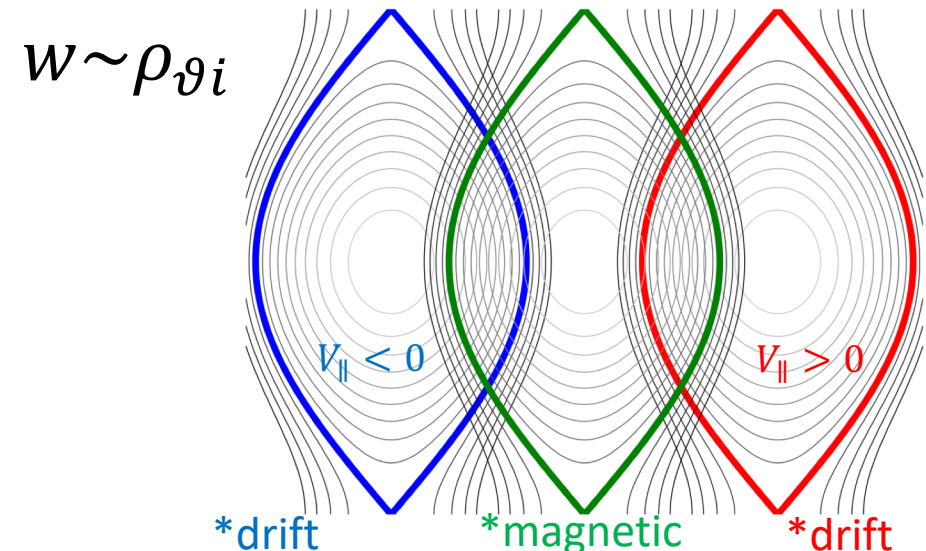
S contours compared to island flux contours

- Passing constant S contours are same topology as island, but radially shifted by $O(\rho_{\vartheta})$, ρ_{ϑ} is the Larmor radius calculated with the poloidal field
- Constant S drift islands shift in opposite directions for opposite $V_{\parallel}$

- large islands

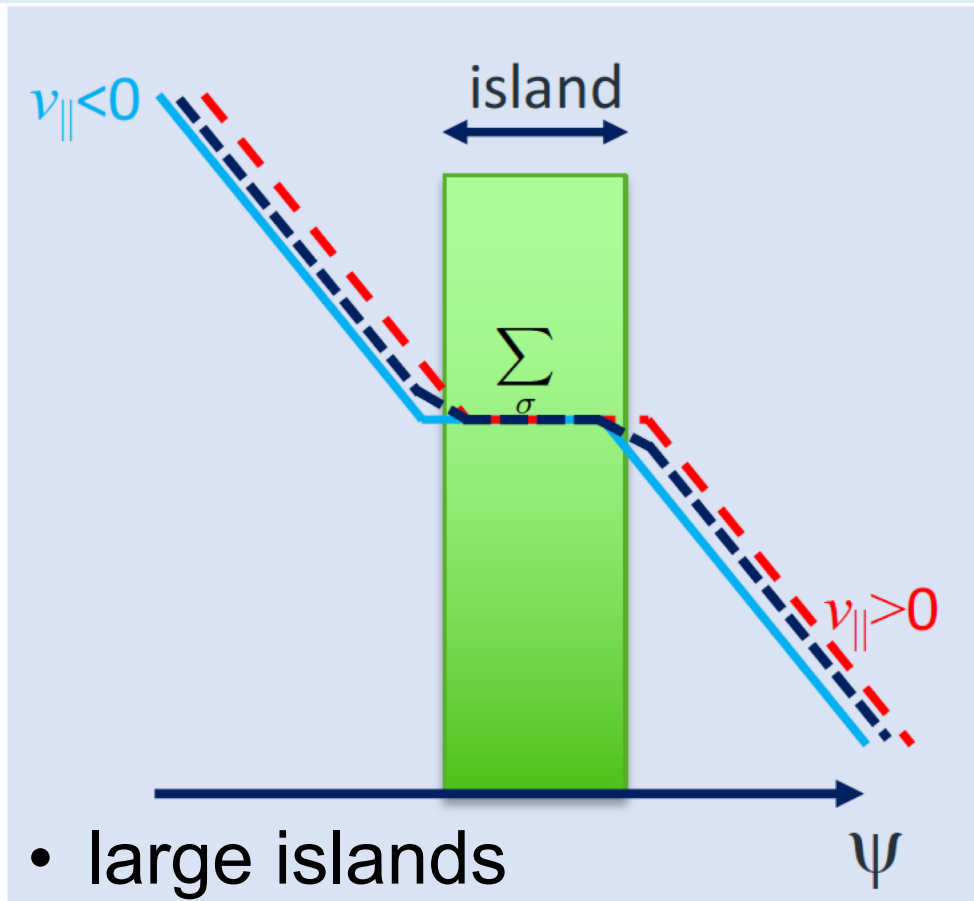
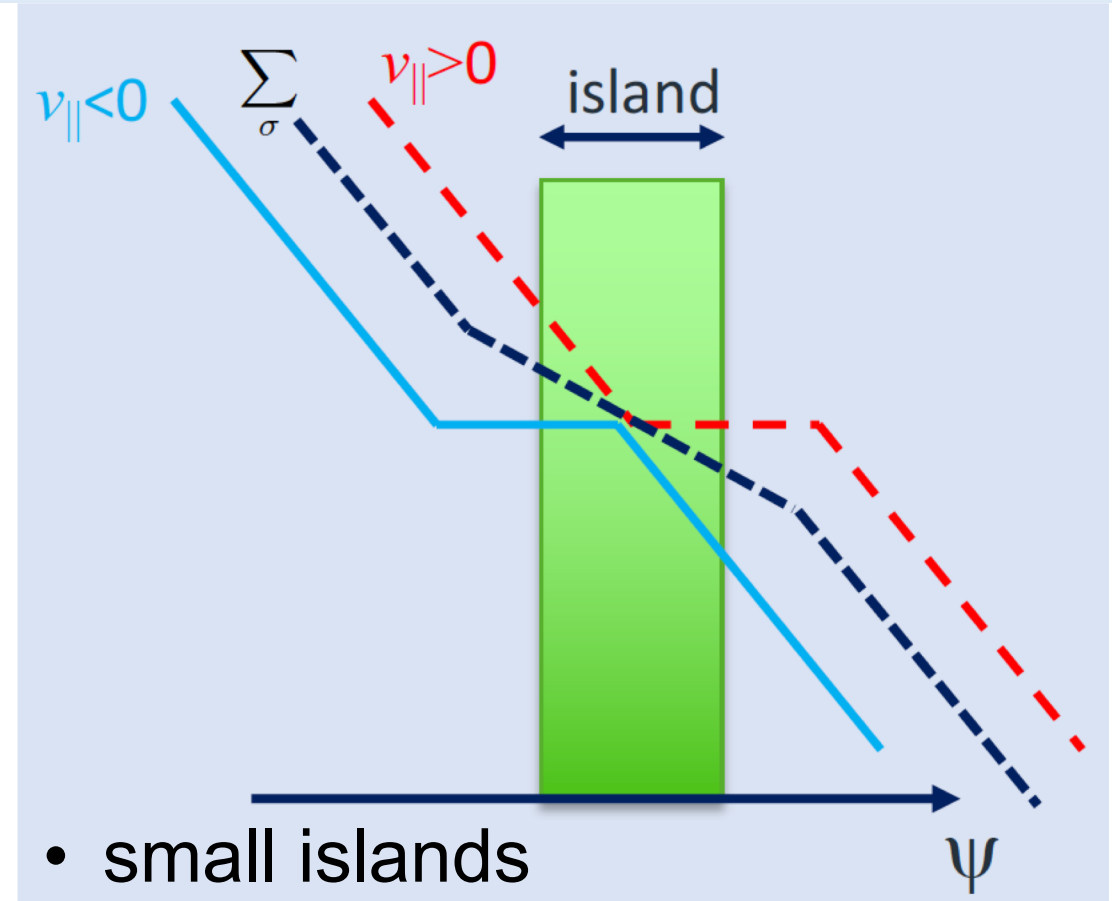


- small islands close to threshold



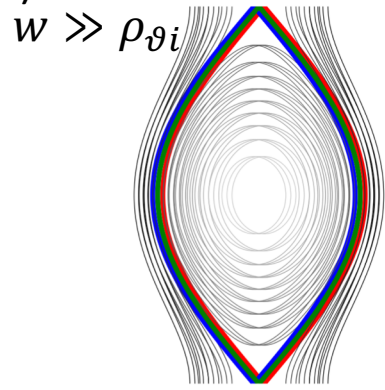
Distribution function across drift islands

- distribution function is found to be flattened across drift islands rather than magnetic island
- This restores density gradient across small islands

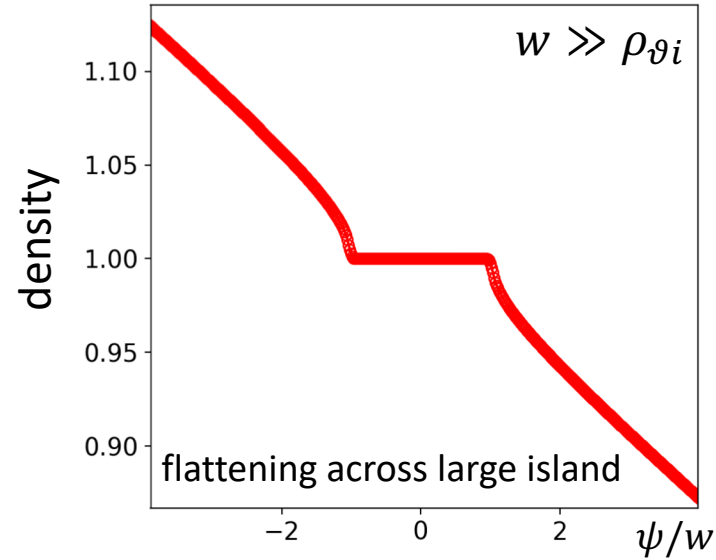


Density and parallel flows

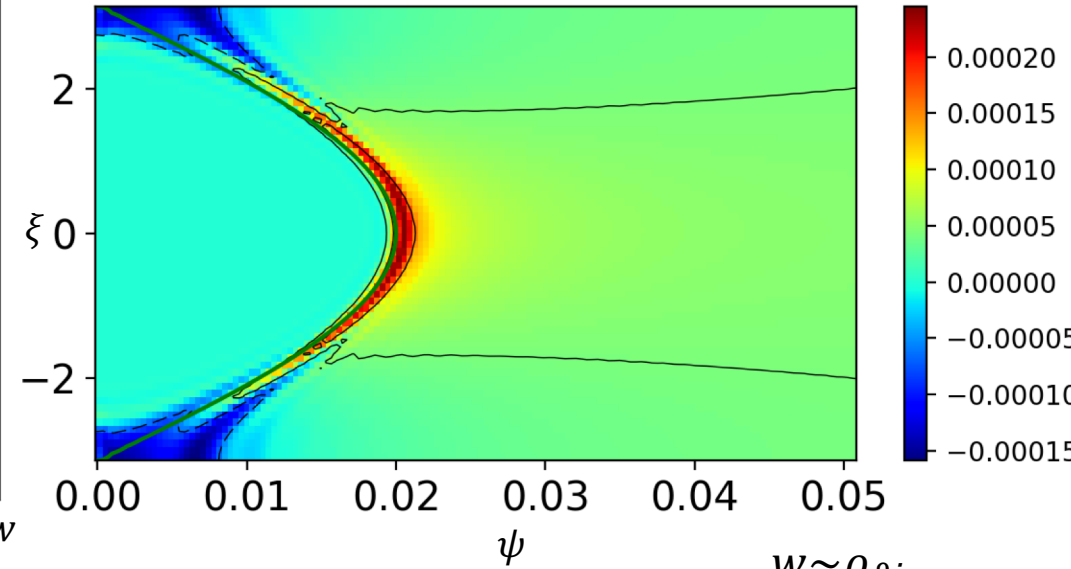
- large islands (matches conventional model)



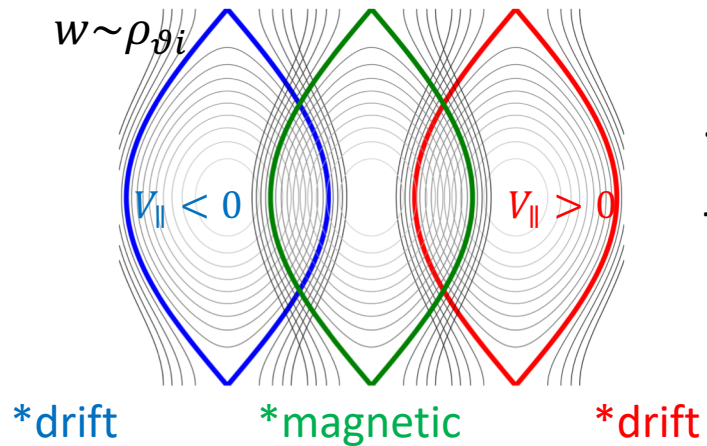
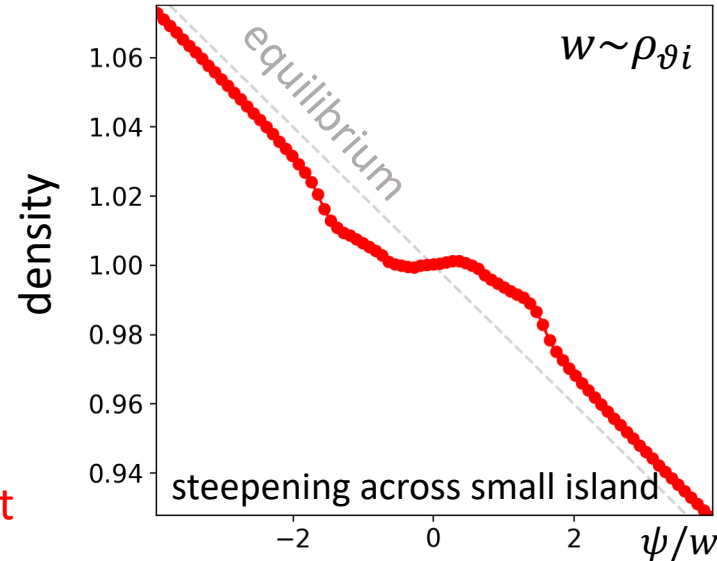
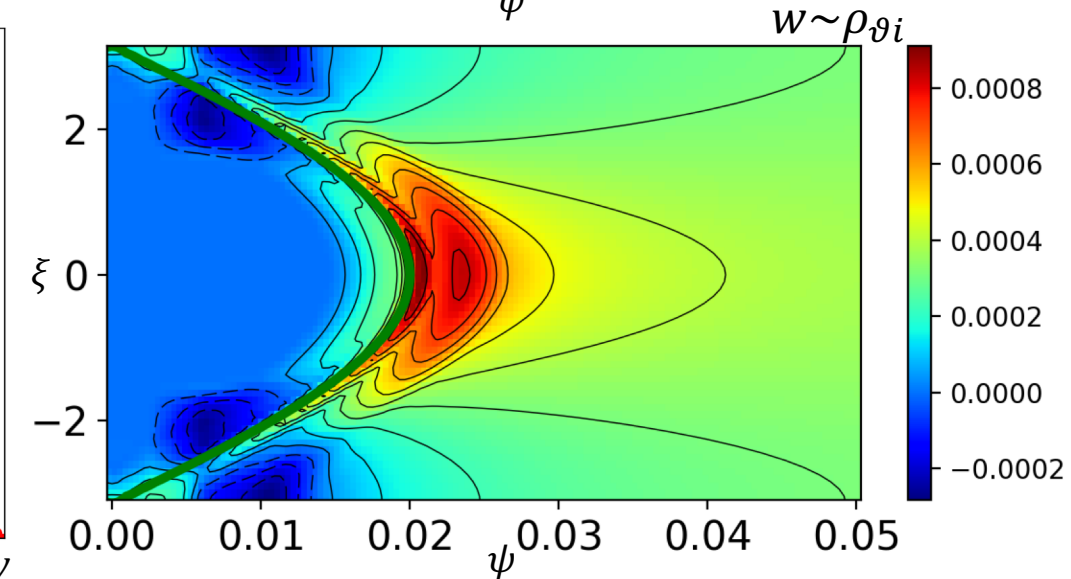
- density profiles:



- parallel current profiles: $w \gg \rho_{\vartheta i}$



- small islands close to threshold



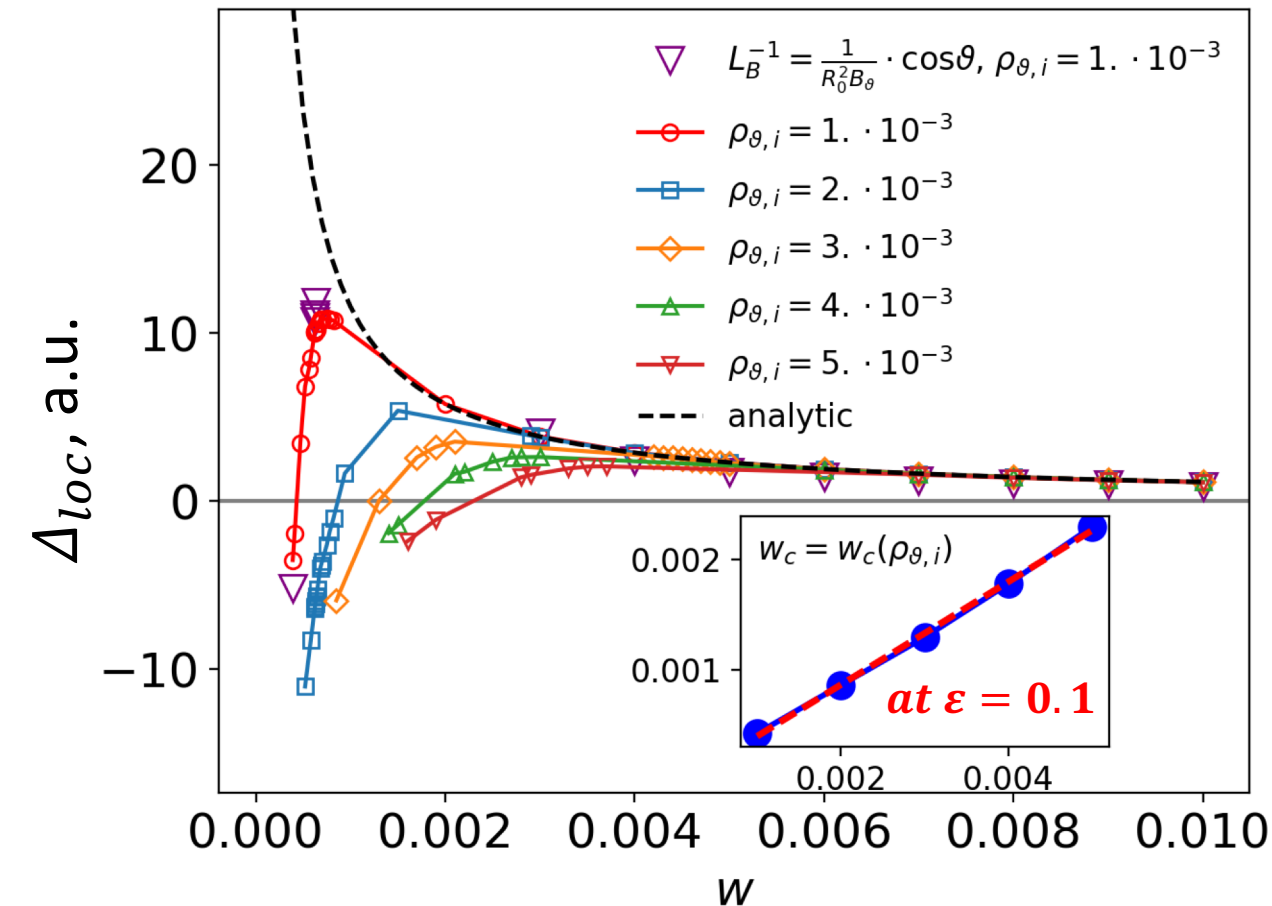
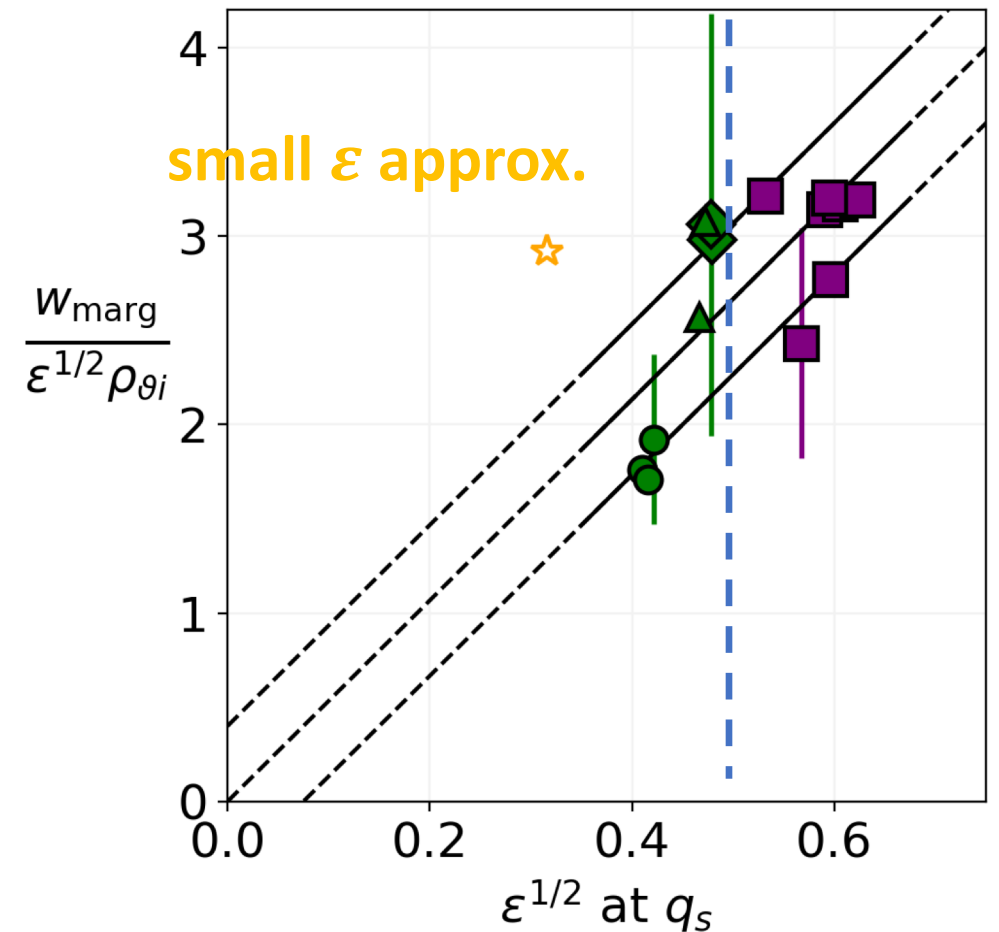
Neoclassical Tearing Mode threshold

$$\frac{dw}{dt} = \frac{1}{\mu_0 R} \Delta' \tilde{\psi} - \underbrace{\int_{\mathbb{R}} d\psi \int_{-\pi}^{\pi} d\xi \bar{J}_{\parallel} \cos n\xi}_{\Delta_{loc}}$$

$$w_c = 2.92 \rho_{bi} @ \delta = 0 \quad \kappa = 1$$

- 2/1 NSTX, $q_{95} = 8$
- ◆ 2/1 DIII-D, $q_{95} = 4$
- ▲ 3/1 DIII-D, $q_{95} = 7$
- 2/1 DIII-D, $q_{95} = 7$
- ☆ 2/1 RDK-NTM, $\varepsilon \ll 1$ approx.
- ★ 2/1 RDK-NTM, finite ε

- w_c is defined as $\Delta_{loc} = 0$



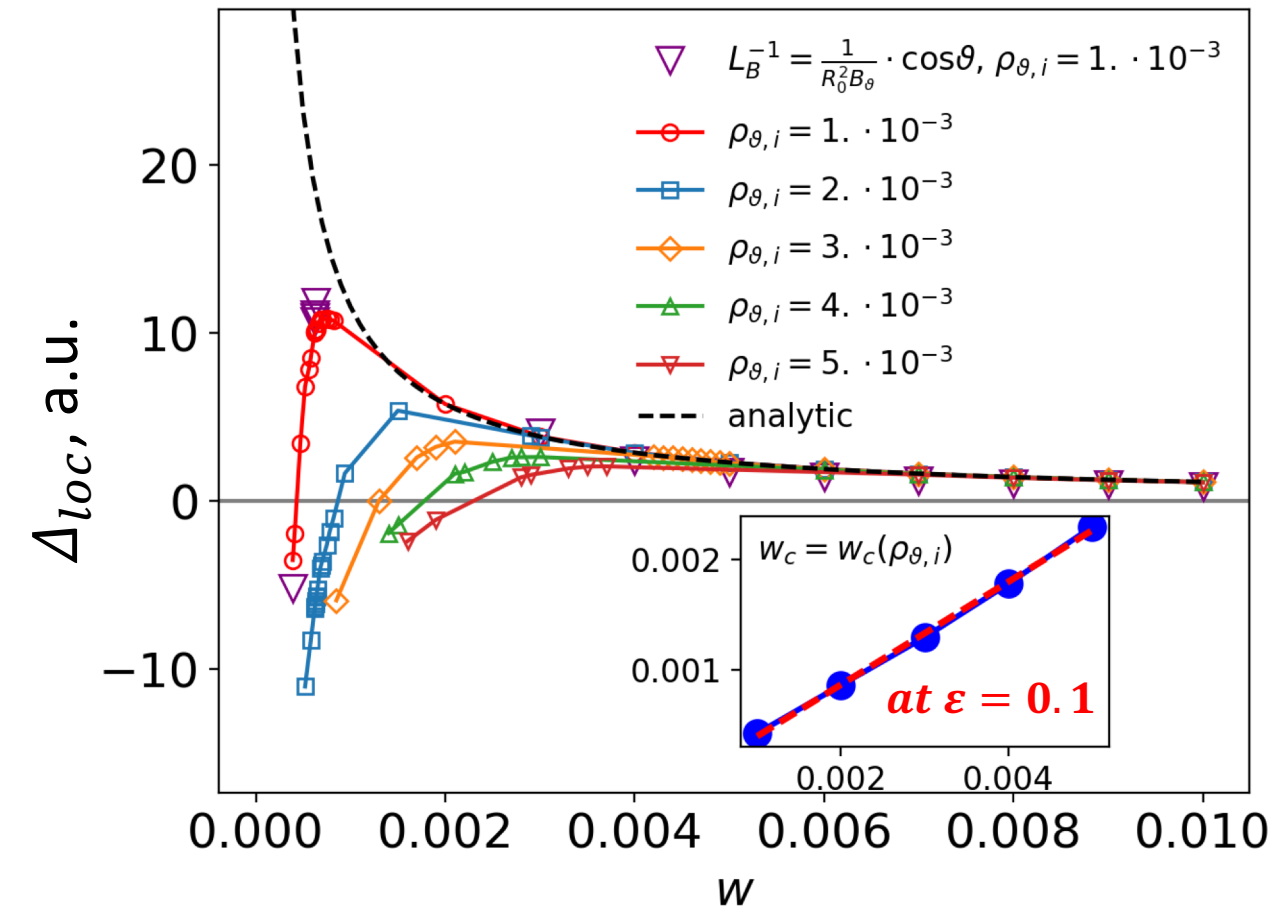
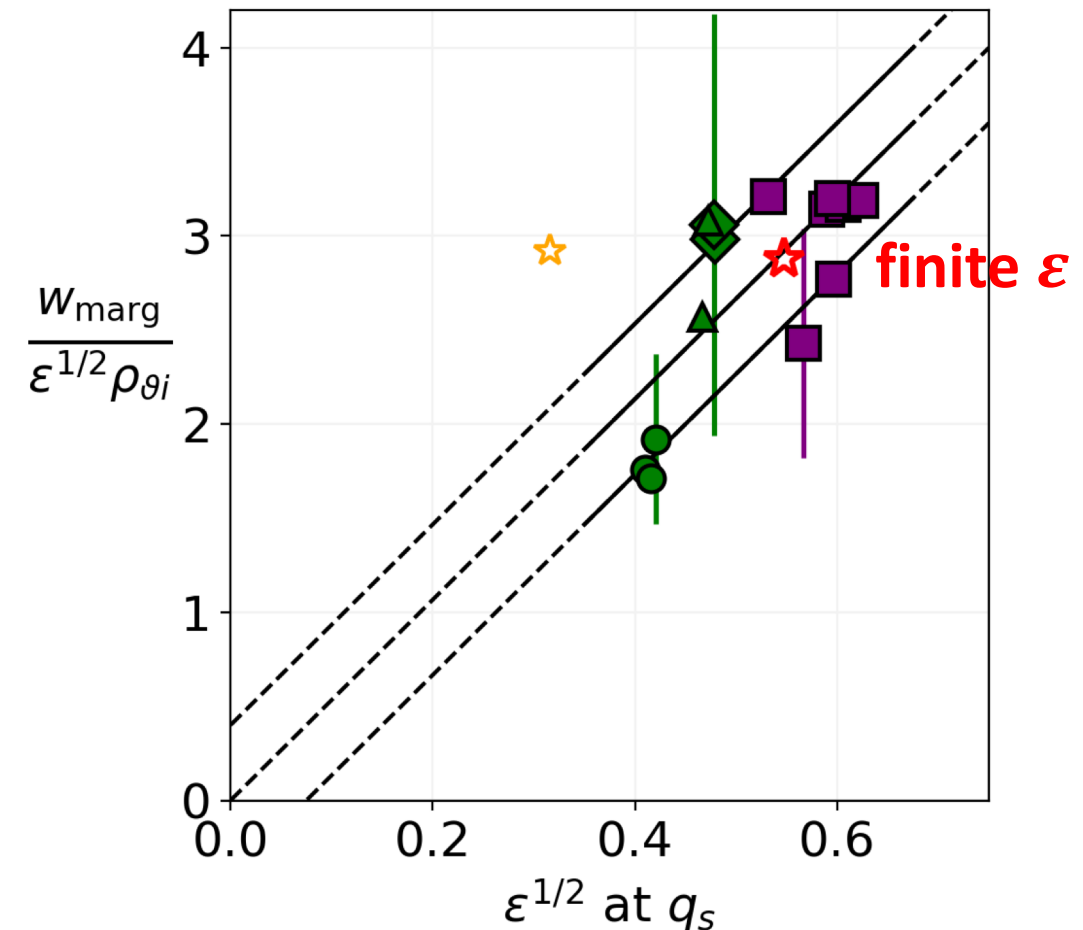
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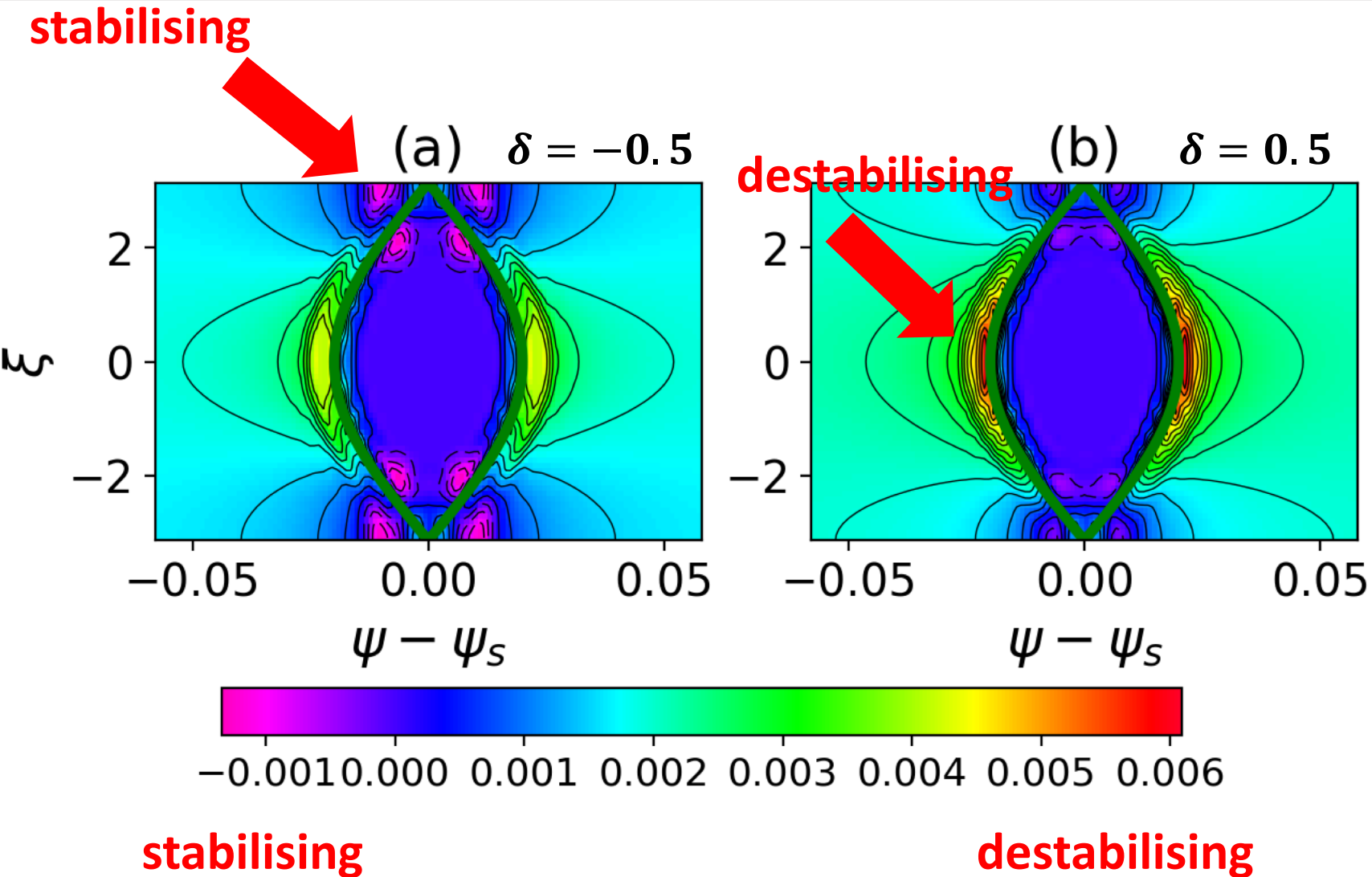
**$w_c = 2.83 \rho_{bi}$ @
 $\delta = 0.4, 0.7 \quad \kappa = 2$**

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- ◆ 2/1 DIII-D, $q_{95} = 4$
- ▲ 3/1 DIII-D, $q_{95} = 7$
- 2/1 DIII-D, $q_{95} = 7$
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- w_c is defined as $\Delta_{loc} = 0$



Triangularity effects



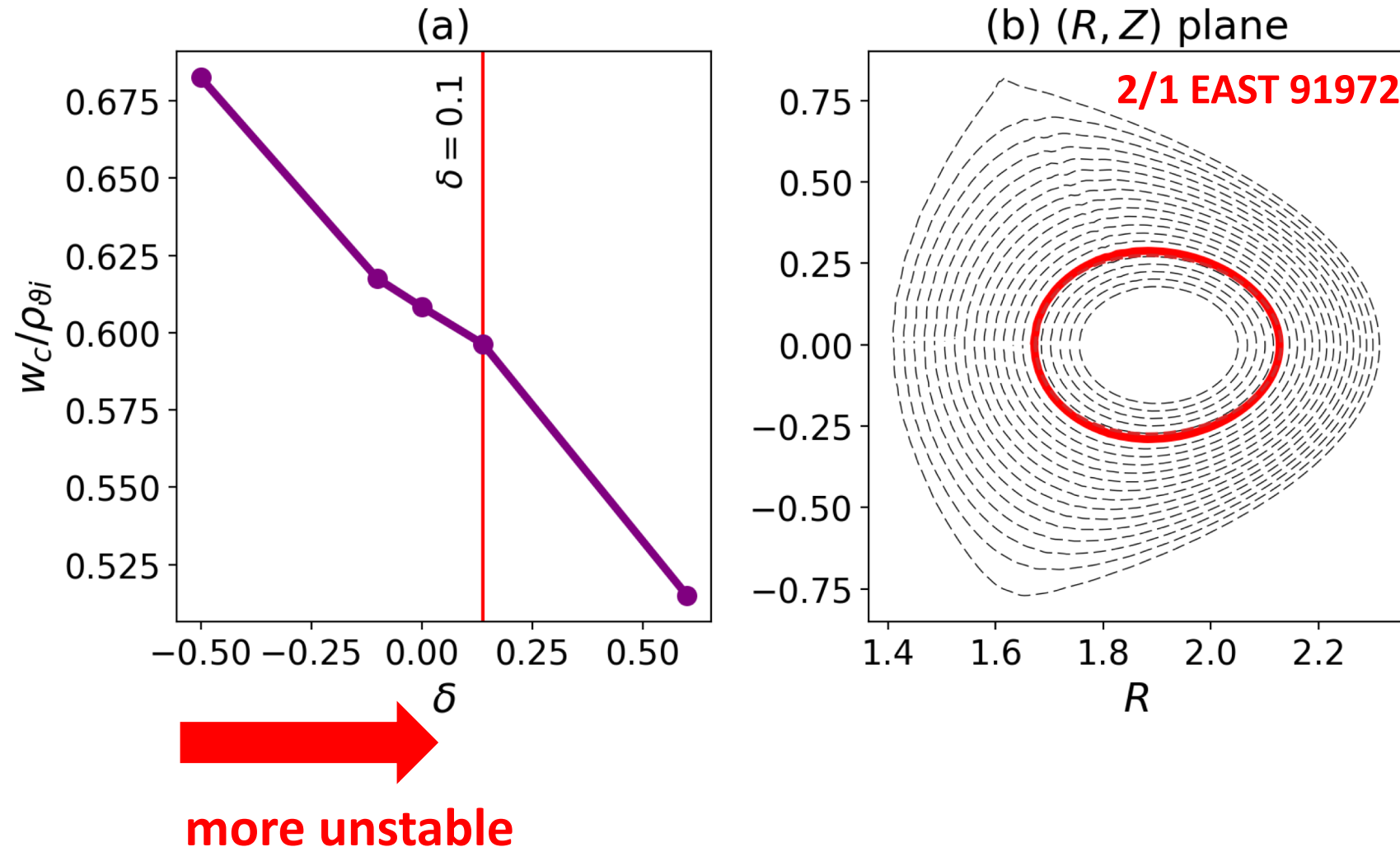
- negative triangularity amplifies negative flows, making them spread further into island region, enhancing drift island effects

- positive triangularity amplifies positive flows

- higher triangularity plasma is more prone to NTMs

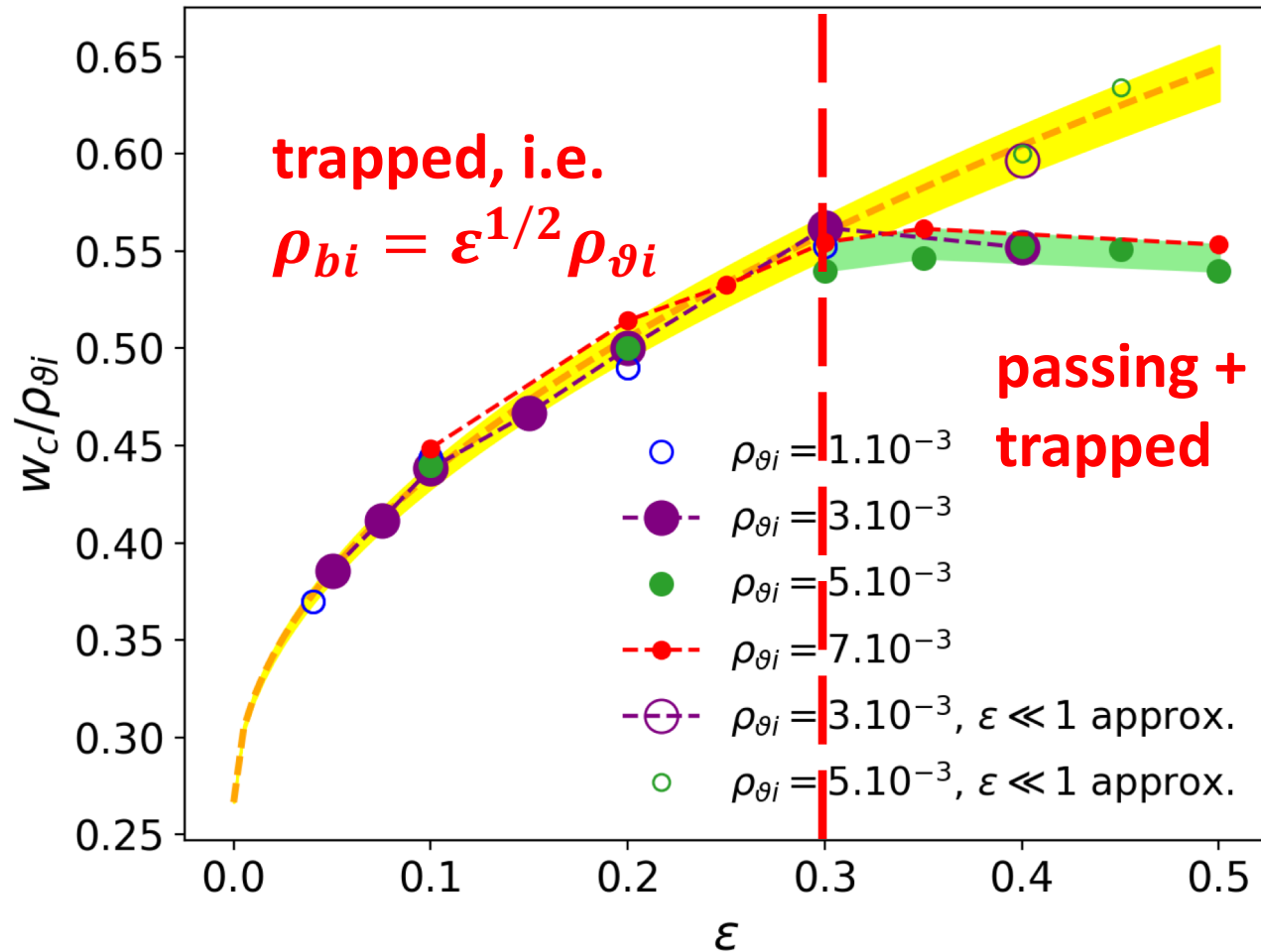
Triangularity effects

NTM threshold vs. triangularity δ



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- higher triangularity plasma is more prone to NTMs

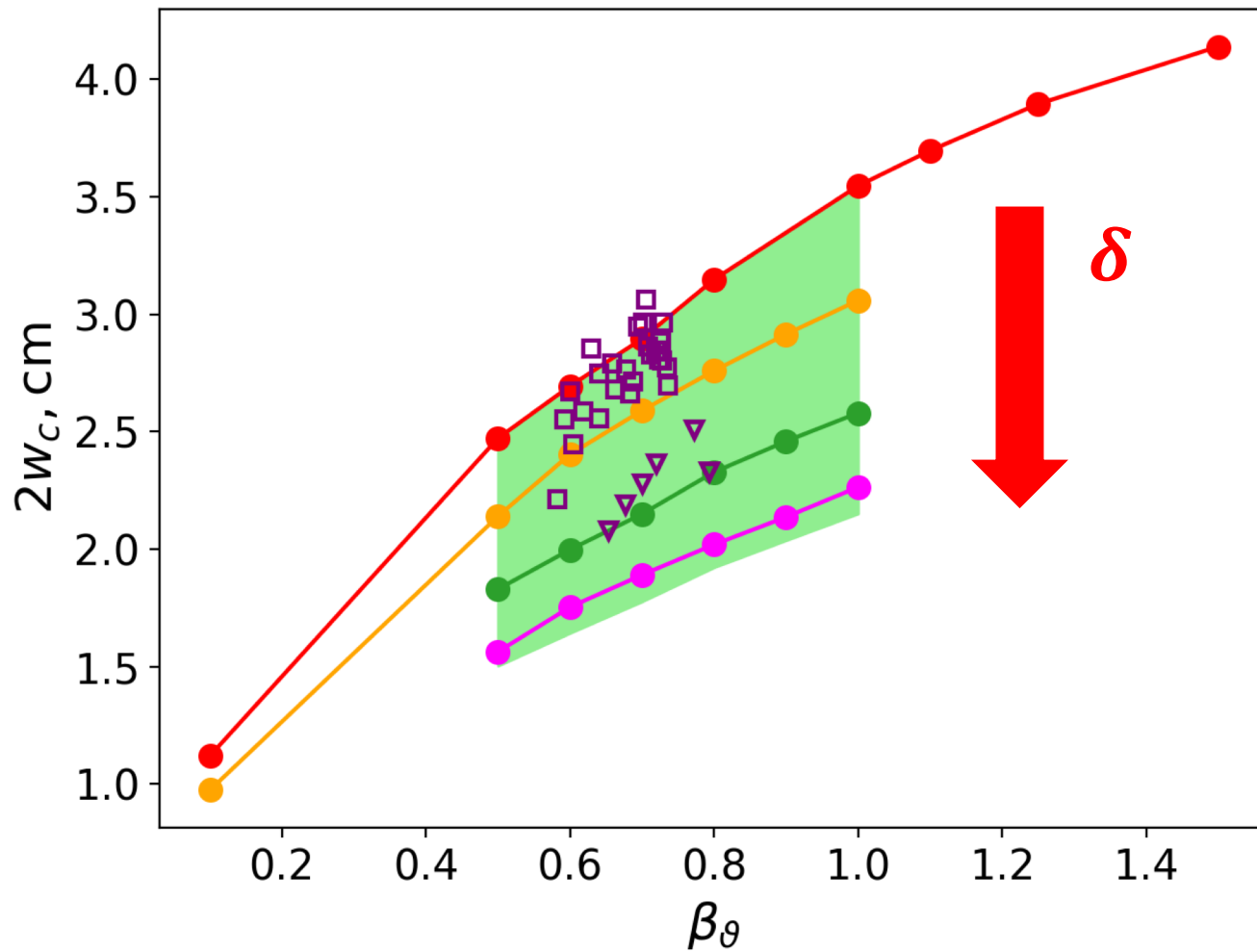
Finite aspect ratio effects



Submitted to Nucl. Fusion (2022)

- ion density gradient supported by **passing** ion drift island effects (key parameter is $\rho_{\theta i}$)
- electron density gradient then supported by quasi-neutrality
- trapped particle dynamics is introduced via
 - trapped-passing boundary
 - perturbed electrostatic potential
- **trapped** particle parameter is ρ_{bi}
- scaling up to $\epsilon = 0.3$ indicates this

Poloidal beta effects



Submitted to Nucl. Fusion (2022)

- w_c as a function of β_θ
- Experimental data is based on 2/1 NTM triggered by RMPs (EAST 91972 5300ms)
- Theory: based on drift kinetic solution at different triangularity

- RDK-NTM, $\delta = 0$
- RDK-NTM, $\delta(r_s)$
- RDK-NTM, $\delta = 0.4$
- RDK-NTM, δ (LCFS)
- EAST, $B_\phi < 0$
- ▽ EAST, $B_\phi > 0$

Summary

- self-consistent drift kinetic theory of magnetic islands, valid for $w \sim \rho_{\vartheta i}$ and small ITER relevant collisions
- particle distribution function is flattened across drift islands, which are radially shifted by a value $\propto \rho_{\vartheta i}$ compared to magnetic island
- $\nabla n / \nabla T$ is not removed inside the magnetic island of $w \sim \rho_{\vartheta i}$:
 - ion density gradient is restored due to passing (ion) drift island effects
 - perturbed electrostatic potential restores electron density gradient (plasma is quasi-neutral)
 - perturbed electrostatic potential and trapped-passing boundary layer ensure coupling to trapped particles
- This suppresses NTM drive when w is small
- model captures realistic tokamak geometry

- finite aspect ratio effects: $w_c/\rho_{\vartheta i} \propto \varepsilon^{1/2}$ up to $\varepsilon = 0.3$
- at $\varepsilon = 0.3$, finite aspect ratio effects play a role:
 - $\rho_{bi} = \varepsilon^{1/2} \rho_{\vartheta i}$ is no longer valid
- higher triangularity competes with stabilising drift island effects, i.e. higher triangularity plasma is more prone to NTMs
- effects of plasma elongation are not found to be significant (note: drift kinetic theory assumptions)

Future work

- more accurate treatment of separatrix layer
- quantify effects associated with energetic particles
- relax small poloidal field assumption (extensions to gyrokinetics)
- identify experimental tests